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Numerical modeling of development of fracture in anisotropic composite materials at low-velocity loading

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Abstract The problem of normal interaction of the steel isotropic compact cylindrical projectile with the orthotropic plate on ballistic limit in range of velocities of impact from 50 to 400 m/s is considered. Target material is as organoplastic with initial orientation of mechanical properties, and the material, which properties received by turn on 90° relative to the axis *OY* of an initial material. Fracture of targets is investigated; the comparative analysis of efficiency of their protective properties depending on orientation of elastic and strength properties of an anisotropic material is carried out. The task is solved numerically, using the method of finite elements in three-dimensional statement. The behavior of a material of the projectile is described by elastic–plastic model; the behavior of anisotropic material of the target is described by elastic–brittle model with various ultimate strength limits on pressure and tension.

Introduction

Reactions of isotropic and anisotropic materials to external loading have essential quantitative and qualitative distinctions. And, if at static loadings such distinctions are caused by that in an anisotropic material such characteristics of a material as coefficients of elasticity and strength parameters depend on a direction, at dynamic loadings the additional factor influencing an intense-deformed condition of an anisotropic material, is dependence on a direction of velocity of stress waves propagation.

In spite of the fact that the newest materials with preselected orientation of properties have wide application as constructional, quantity of the papers, devoted to research of their properties at dynamic loadings, is extremely insignificantly. The analysis of behavior of such materials is spent, as a rule, with use of engineering techniques and allows to receive approximate estimates of integrated parameters for the conditions supposing fall of dimension of a problem from three (behavior of anisotropic materials, as a rule, three-dimensional) to two. Similar cases are limited to axisymmetric influence on a transtropic material. However, such key factors as dynamics of destruction, the comparative analysis of behavior of materials with various symmetry of properties, evolutions of wave processes, influence of orientation of properties, which can become defining at dynamic processes, remain behind frameworks of similar techniques.

In work features of deformation and destruction of fragile anisotropic materials at shock loading by steel projectiles are considered. Influence of turn of elastic and strength properties of organoplastic on penetration of targets in a range of velocities 50–400 m/s is investigated. Various strength of a material on a tension and pressure is considered. Research of behavior of a material at low-velocity interaction allows to retrace laws of origin and progress of a fracture in anisotropic material.

Equations of mathematical model

The system of the equations describing non-stationary adiabatic movements of the compressed media in the

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Cartesian coordinate system *XYZ*, includes following equations [1]:

– continuity equation

$$\dot{\rho} + div\rho\vec{\upsilon} = 0$$

– motion equation

$$\rho\dot{u} = \sigma_{xx,x} + \sigma_{xy,y} + \sigma_{xz,z}$$
$$\rho\dot{v} = \sigma_{yx,x} + \sigma_{yy,y} + \sigma_{yz,z}$$
$$\rho\dot{w} = \sigma_{zx,x} + \sigma_{zy,y} + \sigma_{zz,z}$$

– energy equation

$$\dot{E} = \frac{1}{\rho}\sigma_{ij}e_{ij};\ i,j = x,y,z$$

here ρ is density of media; $\vec{\upsilon}$ is velocity vector, u, v, w are components of velocity vector on axes x, y, z accordingly; σ_{ij} is components of a symmetric stress tensor; E is the specific internal energy; e_{ij} is components of a symmetric strain rate tensor; the point over a symbol means a time derivative; a comma after a symbol is a derivative on corresponding coordinate.

The behavior of the aluminum isotropic cylinder at high-velocity impact is described by elastic–plastic media, in which communication between components of strain velocity tensor and components of stress deviator are defined by Prandtl–Reuss equation [2, 3]:

$$2G\left(e_{ij} - \frac{1}{3}e_{kk}\delta_{ij}\right) = \frac{DS^{ij}}{Dt} + \lambda S^{ij},\ (\lambda \geq 0)$$

$$\frac{DS^{ij}}{Dt} = \frac{dS^{ij}}{dt} - S^{ik}\omega_{jk} - S^{jk}\omega_{ik},$$

where $\omega_{ij} = \frac{1}{2}\left(\nabla_i\upsilon_j - \nabla_j\upsilon_i\right)$, G is shear modulus. Parameter $\lambda = 0$ at elastic deformation and at elastic ($\lambda > 0$) is defined by means of a Mises condition:

$$S^{ij}S_{ij} = \frac{2}{3}\sigma_{\mathrm{d}}^2,$$

where σ_{d} is dynamic yield point. The ball part of stress tensor (pressure) is calculated on the Mi-Gruneisen equation as function of specific internal energy E and density ρ:

$$P = \sum_{n=1}^{3} K_n\left(\frac{V_0}{V} - 1\right)^n\left[\frac{1 - K_0\left(\frac{V_0}{V} - 1\right)}{2}\right] + K_0\rho E,$$

where K_0, K_1, K_2, K_3 are the constants of material.

The behavior of an anisotropic material of targets is described within the limits of elastic–fragile model [1, 4]. Before fracture components of a stress tensor in a target material were defined from equations of the generalized Hooke's law which e_{ij} have been written down in terms of strain rate:

$$\dot{\sigma}_{ij} = C_{ijkl}e_{kl},$$

where C_{ijkl} is the elastic constants.

Thus components of a tensor of elastic constants possess, owing to symmetry of stress tensors and strain tensors and presence of the elastic potential, following properties of symmetry:

$$C_{ijkl} = C_{jikl} = C_{ijlk} = C_{ijlk}, C_{ijkl} = C_{klij}$$

At transition to another, also orthogonal, coordinate system, elastic constants will be transformed by equations:

$$C'_{abcd} = C_{ijkl}q_{ia}q_{jb}q_{kc}q_{ld}$$

where q_{ij} is cosine of the angle between corresponding axes i and. In three-dimensional space transformation of the component of a tensor of the fourth rank demands summation of the compositions, containing as multipliers 4 cosines of angles of rotation of axes.

Fracture of an anisotropic material is described within the limits of model [1, 4] with use of Wu's fracture criterion [5] with various ultimate strengths of pressure and tension. This criterion, which has been written down by scalar functions from components of a stress tensor, has the following appearance:

$$f(\sigma_{ij}) = F_{ij}\sigma_{ij} + F_{ijkl}\sigma_{ij}\sigma_{kl} + \ldots \geq 1, \quad i,\ j,\ k,\ l\ = 1, 2,\ 3.$$

here F_{ij} and F_{ijkl} are the components of tensor of the second and the fourth rank respectively, and obey transformation laws:

$$F'_{ab} = F_{ij}q_{ia}q_{jb},\ F'_{abcd} = F_{ijkl}q_{ia}q_{jb}q_{kc}q_{ld}.$$

Components of tensors of strength for criterion are defined by following equations:

$$F_{ii} = \frac{1}{X_{ii}} - \frac{1}{X'_{ii}},\ F_{iiii} = \frac{1}{X_{ii}X'_{ii}}$$

$$F_{ij} = \frac{1}{2}\left(\frac{1}{X_{ij}} - \frac{1}{X'_{ij}}\right),\ F_{ijij} = \frac{1}{4X_{ij}X'_{ij}}, \quad i \neq j$$

where X_{ii}, X'_{ii} are limits of strength on pressure and tension along the direction i; X_{ij}, X'_{ij} are shear strength along the two opposite directions with $i \neq j$. Coefficients F_{1122}, F_{2233}, F_{3311}, are defined at carrying out the experiments on biaxial tension in planes 1–2, 2–3, 1–3 accordingly. The remained coefficients are defined similarly at combined stressing in corresponding planes.

It is supposed that fracture of anisotropic materials in the conditions of intensive dynamic loads occurs as follows:

– if strength criterion is violated in the conditions of pressure ($e_{kk} \leq 0$) the material loses anisotropy of properties, and its behavior is described by

hydrodynamic model, thus the material keeps its strength only on pressure; the stress tensor becomes in this case spherical $\sigma_{ij} = -P$;
– if the criterion is violated in the conditions of tension ($e_{kk} > 0$), the material is considered completely fractured, and components of a stress tensor are appropriate to be equal to zero.

Adequacy of the model

The series of test calculations for check convergence of the solution (independence of the solution from a spatial interval) has been passed. Dependence of the σ_{xx} component in the central point of a target from total number of elements N_E in a calculating grid was considered. The received curve shows fast convergence of the decision at a grid compaction (Fig. 1).

In connection with a considerable quantity of works on a problem of impact on rigid target, numerical experiments for check of the numerical methods have been made. The problem about normal impact of the cylindrical projectile with length $L_0 = 23.47$ mm and diameter $D_0 = 7.62$ mm on rigid target with initial velocity v_0 is considered. Material of the projectile—steel of ST3 mark. In the Table 1 the received results of calculations in comparison with experiment and the data of calculations from Wilkins [6] study on residual length of the drummer L are presented at various velocities of impact. In the Table 1 δ is the relative divergence between spent calculations and experiments.

For check of adequacy of model the number of comparisons of numerical calculations with experimental data has been spent [7]. Table 2 shows the results of experiments and calculations at the interaction of the steel projectile with mass of 20 g with the glass–fiber plastic isotropic targets ($\rho_0 = 1930$ kg/m^3). The following symbols are introduced into the table: h is the target thickness, v_0 is the initial

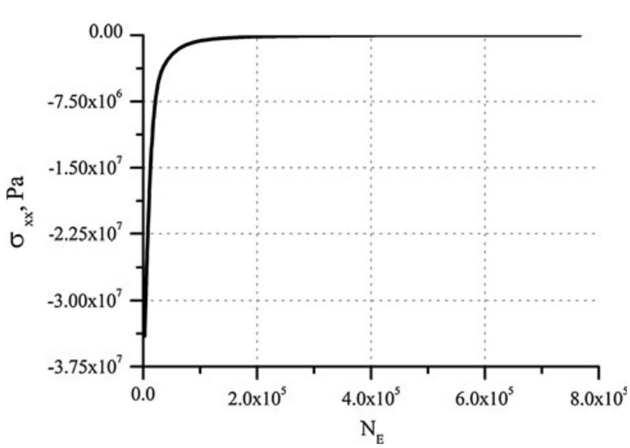

Fig. 1 Convergence of the solution

Table 1 Comparison with experiments and calculations

v_0 (m/s)	Experiment $\left(\frac{L}{L_0}\right)$	Wilkins [6] $\left(\frac{L}{L_0}\right)$	Calculation $\left(\frac{L}{L_0}\right)$	δ (%)
175	0.911	0.911	0.915	0.4
252	0.842	0.842	0.839	0.4
311	0.766	0.766	0.760	0.8
402	0.635	0.667	0.619	2.5

Table 2 Comparison with experiments for isotropic targets

h	v_0 (m/s)	v_1 (m/s)		δ_v (%)
		Experiment	Calculation	
5	992	928	870	6.3
9	1163	1013	950	6.2
14	1064	812	780	4.0

Table 3 Comparison with experiments for anisotropic targets

h (mm)	v_0 (m/s)	v_1 (m/s)		δ_v (%)
		Experiment	Calculation	
26	1054	698	640	8.3
26	1077	695	638	8.2
18	1012	897	836	6.8
18	956	838	792	5.5

projectile velocity, v_1 is the post-perforation velocity of projectile, δv is the relative divergence between post-perforation velocity of projectile in the experiment and calculation. The results on post-perforation velocity of projectile at the interaction of 20-g steel projectile with organoplastic anisotropic targets ($\rho_0 = 1350$ kg/m^3) are presented in Table 3.

Comparison of numerical and experimental results allows to draw the conclusion that the offered model well describes process of penetration of isotropic and anisotropic plates. A deviation of the calculating values of post-perforation velocities from experimental values does not exceed 8.5%.

Formulation of the task

It is considered (Fig. 2) a three-dimensional task of high-speed interaction of compact ($d_0 = h_0 = 15$ mm, diameter of the projectile is equal to its height) cylindrical projectile (area D_1) with target (area D_2). A thickness of a target is 15 mm.

A material of the projectile is isotropic aluminum ($\rho_0 = 2710$ kg/m^3, $\sigma_d = 310$ MPa), a material of targets—orthotropic organoplastic ($\rho_0 = 1350$ kg/m^3). Orientation

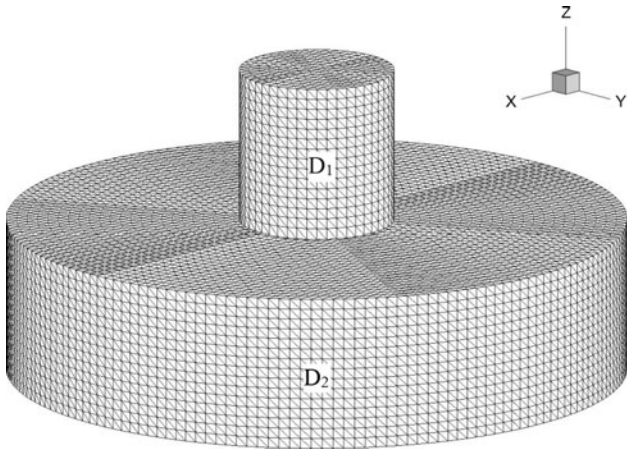

Fig. 2 Three-dimensional formulation of the task

of properties of orthotropic material changes by turn of axes of symmetry of an initial material round an axis on an angle $\beta = 90°$. As an initial material ($\beta = 0°$) of targets it is considered orthotropic organoplastic with the following elastic and strength characteristics: $E_x = 48.6$ GPa, $E_y = 21.3$ GPa, $E_z = 7.14$ GPa, $c_x = 6000$ m/sec, $c_x = 3970$ m/sec, $c_z = 2300$ m/sec, $v_{xy} = 0.28$, $v_{yz} = 0.26$, $v_{xz} = 0.25$, $\sigma_x^p = 2.67$ GPa, $\sigma_y^p = 1.18$ GPa, $\sigma_z^p = 0.39$ GPa, $\sigma_x^c = 0.37$ GPa, $\sigma_y^c = 0.5$ GPa, $\sigma_z^c = 1.94$ GPa, $\tau_{xy} = 0.975$ GPa, $\tau_{yz} = 0.8$ GPa, $\tau_{xz} = 0.607$ GPa. Here E_x, E_y, E_z и c_x, c_y, c_z—modules of elasticity and velocity of sound in corresponding directions; v_{xy}, v_{yz}, v_{xz}—Poisson's ratios; σ_x^p, σ_x^c, τ_{xy}—strength parameters on tension, pressure, and shear. The range of initial velocities of the projectile from 50 to 400 m/sec is investigated. The meeting angle (between a normal to a target and a longitudinal axis of the projectile) is $\alpha = 0°$ (normal impact).

On free surfaces of the projectile and target conditions $\vec{T}_{nn} = \vec{T}_{ns} = \vec{T}_{n\tau} = 0$ are satisfied, on contact surfaces between the projectile and target conditions of sliding without a friction $\vec{T}_{nn}^{+} = \vec{T}_{nn}^{-}$, $\vec{T}_{n\tau}^{+} = \vec{T}_{n\tau}^{-} = \vec{T}_{ns}^{+} = \vec{T}_{ns}^{-} = 0$, $\vec{\upsilon}_n^{+} = \vec{\upsilon}_n^{-}$, are realized. Here $\vec{n}$—a unit vector of a normal to a surface in a considered point, $\vec{\tau}$ и $\vec{s}$—unit vectors, tangents to a surface in this point, $\vec{T}_n$—a force vector on a platform with a normal $\vec{n}$, $\vec{\upsilon}$—a velocity vector. The bottom indexes at vectors $\vec{T}_n$ and $\vec{\upsilon}$ also mean projections on corresponding basis vectors; the sign plus "+" characterizes value of parameters in a material on the top border of a contact surface, the sign minus "–" on bottom.

Discussion

In Figs. 3, 4, 5, 6 configurations of the projectile and targets with distribution of isolines of relative volume of destructions for various velocities of interaction at the moment of time $t = 40$ μs are presented. To the left of a symmetry axis configurations for initial orientation of a material of a target, to the right, for the reoriented material are given.

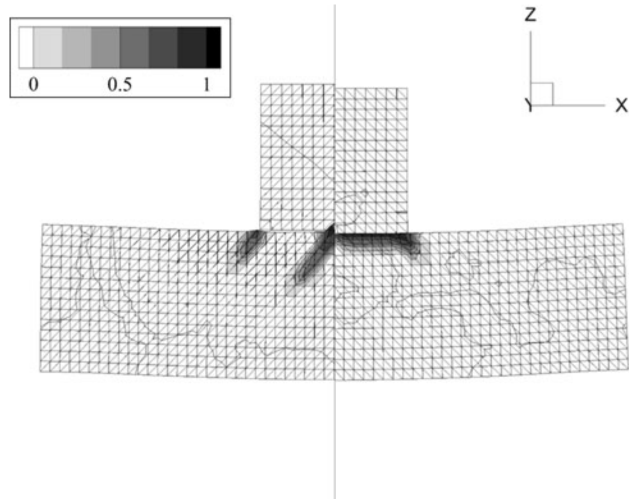

Fig. 3 Relative volume of fractures in target at $v_0 = 50$ m/s and $t = 40$ μs

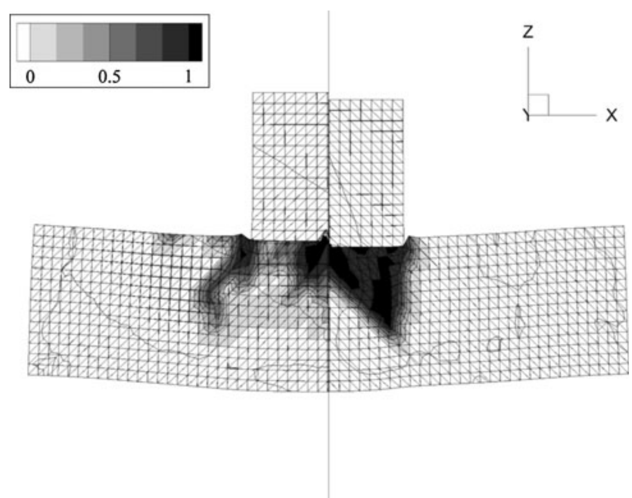

Fig. 4 Relative volume of fractures in target at $v_0 = 100$ m/s and $t = 40$ μs

For a case of initial orientation of properties of organoplastic at velocity 50 m/s (Fig. 3, to the left of a symmetry axis) on an obverse surface of a target on perimeter of the projectile and on a contact surface in the target center the conic zones of fracture focused at an angle 45° to a direction of impact are formed. These zones arise in an initial stage of interaction at the expense of action of tensile stress in the unloading waves extending from an obverse surface of a target and a lateral surface of the projectile. The further development of these zones of fracture is caused by action of tensile stress as a result of

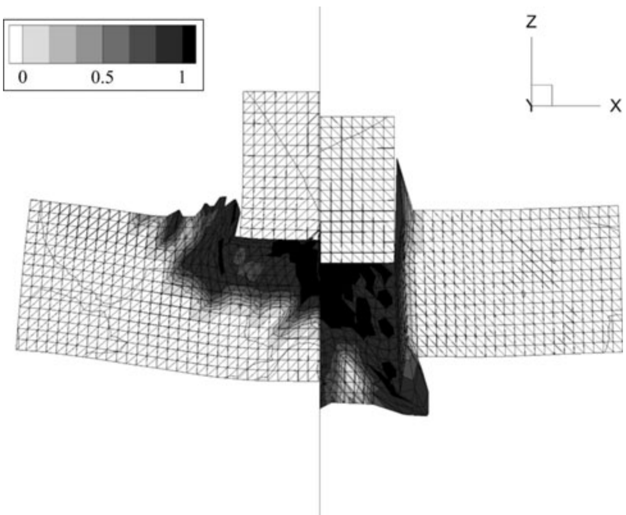

Fig. 5 Relative volume of fractures in target at $v_0 = 200$ m/s and $t = 40$ µs

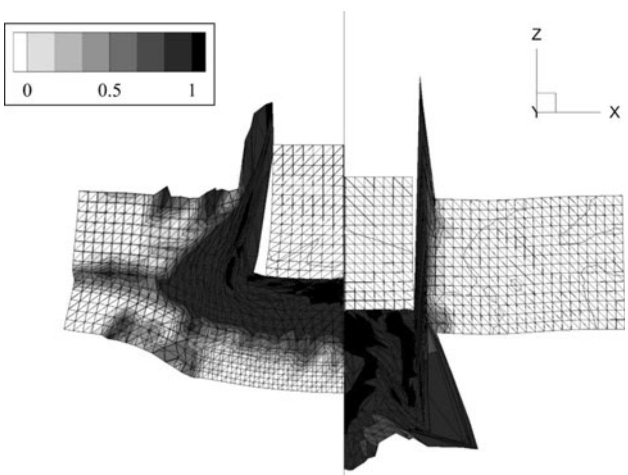

Fig. 6 Relative volume of fractures in target at $v_0 = 400$ m/s and $t = 40$ µs

introduction of the projectile. At initial velocity 50 m/s there is no perforation of a target. To $t = 30$ µs velocity of the projectile reduces to zero and the kickback of the projectile from a target is observed. Values of vertical component of the velocity of the center of projectile weights and a part of the fractured material of a target are presented in Table 4 (at tension D_t and pressure D_p at the moment of time $t = 50$ µs).

In case of the reoriented material (Fig. 3, to the right of a symmetry axis) a picture of development of fracture is qualitative other. In this case, strength of a material on pressure in a direction of axis Z (an impact direction) is minimal. It leads to that the material break in the wave of pressure formed at the moment of impact and extending on a thickness of a target. Penetration of the projectile thus occurs in already weakened material. Though perforation in this case also is not present, the projectile gets deeply, and its full braking is observed in 50 µs. With increase in velocity of impact the volume of areas of fracture grows. At velocity 100 m/s (Fig. 4) fracture areas extend to a greater depth on a thickness of a target. And for an initial material of a target the marked orientation (45°) was kept only by a crack extending from an obverse surface on perimeter of the projectile. The crack located near to an axis of symmetry is not identified any more. It is caused by that with increase in velocity of impact the amplitude of the pressure wave grows—its size is already sufficient for material destruction in the top half of target.

In case of the reoriented material the unloading wave extending from a back surface of a barrier, lowers level of compressive stresses that leads to smaller distribution of fracture area on a thickness near to a symmetry axis (Fig. 4). For velocity 100 m/s also perforation of targets is not observed , thus in case of an initial material velocity of the projectile reduced to zero at 45 µs, in case of the reoriented material—at 6 0µs.

For velocities of impact 200 m/s and above (Figs. 5, 6) it is already observed perforation of targets from both types of materials. But thus the plate from an initial material greater maintains resistance to penetration of the projectile in comparison with a plate from the reoriented material. For example, at initial velocities 200 m/s (Fig. 5) and 400 m/s (Fig. 6) post-perforation velocity of the projectile after perforation of plates from an initial material makes 37 and 187 m/s accordingly, and post-perforation velocity after perforation plates of the reoriented material 125 and 300 m/s. Greater resistance to penetration of the projectile in plates from an initial material is caused by a various picture of fracture which is defined by orientation of elastic and strength properties in relation to external loading. For

Table 4 Velocity of the center of projectile weights and part of the break material in targets

v_0 (m/s)	50		100		200		400	
β	0°	90°	0°	90°	0°	90°	0°	90°
v_z (m/s)	−5.17	4.08	9.05	12.97	49.97	127.6	191.55	303.69
D_t	0.012	0.005	0.056	0.011	0.162	0.128	0.502	0.282
D_p	0.006	0.002	0.043	0.029	0.104	0.021	0.112	0.019

velocities of impact above 200 m/s there is fracture of the reoriented material in the unloading wave extending from a back surface of a target (Figs. 5, 6), that increases volume of the break material in front of the projectile, essentially reducing resistance to penetration. Such dynamics of fracture become clear by various velocities of wave distribution in the initial and reoriented materials.

In an initial material velocity of wave distribution the greatest in a direction of an axis *X*—perpendicular to an impact direction—therefore unloading waves from an obverse surface of a target and a lateral surface of the projectile lower stresses in a pressure wave to its exit on a back surface that does not lead to material destruction in a pressure wave in the bottom half of plate and an unloading wave from a back surface of the barrier having small amplitude at the expense of easing of a pressure wave.

In the reoriented material velocity of distribution of waves is maximal in a direction of axis *Z*, therefore the pressure wave loses energy only on fracture of a material and being reflected from a back surface by an intensive unloading wave breaking a material.

Conclusions

The quantitative and qualitative analysis of fracture of anisotropic plates of a final thickness at low-velocity impact on ballistic limit is carried out. It is established that formation and a direction of development of fracture zones in a target is defined by orientation of elastic and strength properties of an anisotropic material in relation to an impact direction. Depending on orientation of properties, development of the conic cracks caused by combined action of tensile stresses in waves of unloading and at the expense of penetration of the projectile, or fracture of material in a pressure and unloading wave is probable.

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